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14. ABSTRACT
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The effective description of separated flow can potentially lead to a fast simulation-algorithm for aerodynamic computation. Our estimate predicts that this semianalytical scheme will compute the lift and drag on an aerodynamic body in less than 0.1sec with less than 1\% relative error. This is more than hundredfold increase over current simulation-efficiency.
The enhanced efficiency will enable hitherto impossible exploration of new designs for maximization of the lift to drag ratio. In the future, this will revolutionize aviation technology by the development of bio-inspired aviation mechanism and other novel systems. Such improvements will help in energy-savings and pollution control by reducing fuel consumptions.

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Project report: Efficient simulation and novel modeling by using generic three-dimensional exact solutions to analyze transport dynamics in turbulent vortices

I. INTRODUCTION

Flow-separation from a rigid body is an unsolved problem in theoretical fluid mechanics in spite of the maturity of this field. Classical boundary-layer theory cannot provide a complete description and quantitative estimates for such phenomenon. For example, it is not possible to determine the vortical structures in separated flows by using boundary-layer equation in the wake region [1–7]. Similarly, there is no easy way to predict the strength of the eddies without extensive computations [8, 9]. Unfortunately, subsequent theoretical developments, like triple-deck theory [10–13] or other viscous-inviscid analysis [14–21] have not been able to overcome these deficiencies.

The lack of mathematical understanding of flow-separation is a severe hindrance in aerodynamic analysis. In absence of an analytical method, the brute force computation is the only way to simulate a realistic aerodynamic system [22–28]. For example, the lift and the drag on an object in separated flow can be computed only by numerically solving the full Navier-Stokes equation over a large domain. This is potentially an inefficient approach. Despite increase in computer speed over the years, such inefficiency restricts aviation technology from transformative changes which require exploration of hundreds of thousand cases over wide variety of design parameters.

Recent aspirations in aviation science demand a remedy from these limitations. A fuel-efficient and noiseless aviation device based on bird-like flight can produce ten times more lift to drag ratio [29–33], and can be immensely important for surveillance and transport purposes. This technology is an active topic of research in aerodynamics. However, such bio-inspired designs require vast exploration over geometric shapes and sequences of motion to maximize lift to drag ratio. Moreover, to enhance lift, bird-wings allow cross-flow through it from high pressure to low pressure region during a selective period of the motion [34, 35]. Such mechanism is only possible when a perforated aerodynamic body is considered. The optimum design of perforated wings can be possible only if millions of possibilities are accounted for. Existing numerical methods are not adequate for this purpose. Our recent discoveries (please see III—VI) show that this issue can be addressed by a new theory and a fast algorithm where the computation cost is reduced by orders of magnitudes.

II. BACKGROUND

Aerodynamics is an active field of research for over a century. Modern aerodynamics originating from Prandlt's boundary-layer theory acquired maturity over a long time by many theoretical [36–39], experimental [40–45] and numerical [46–53] studies. The detailed description of these works is beyond the scope of the project. Hence, in the subsequent text, we focus on existing theories and computations directly relevant to our investigation on flow-separation.

The rudimentary insight into flow-separation was first provided by Prandlt who suggested that boundary-layer separates from a point where wall-shear stress vanishes due to the action of an adverse pressure gradient. Unfortunately, several numerical studies [54–57] reported difficulties in obtaining the boundary-layer solution near the separation point characterized according to Prandlt. Later, these difficulties were attributed to Goldstein singularity which has been the focus of numerous mathematical investigations [58–62].

The aforementioned studies indicate peculiar flow-behavior at the point of separation. A number of theories like the triple-deck theory [10–13] can account for the characteristics near the separation point. Also, to this end, other viscous-inviscid models have been used [14–21]. However, though these theories are effective in explaining the flow-features in the vicinity of the separation point, they cannot efficiently compute the global velocity field in the entire flow-domain.

In the past, various aspects of structures in separated flows have also been analyzed [63, 64]. For example, Batchelor proved the approximate uniformity of the vorticity in recirculating wake-structures and proposed a solution scheme based on that. All these studies, however, failed to provide a universal aerodynamic algorithm for global flow-solution with separated boundary-layer.

In absence of a complete mathematical theory of flow-separation, aerodynamic computations are generally done by solving full Navier-Stokes equation [65–70] with standard numerical techniques. These schemes include traditional approaches like finite element [71–75] and finite difference methods [76–82] as well as efficient new methodologies like vortex method[83–96] or spectral method[97–107] which can simulate vortical structures in separated fields particularly well.

The majority of these schemes generally require at least a few minutes to solve for the flow-field and to find lift to drag ratio on a particular aerodynamic body. Though the computation time seems impressive, the current state of the art is inadequate for exploration of new designs with numerous design parameters. Such endeavor needs consideration of hundreds of thousand possibilities which cannot be taken into account without drastic decrease in individual simulation-time. Our mathematical theory for flow-separation serves this purpose by providing an algorithm which takes an estimated 0.1second for solving similar problems with similar accuracy.

III. ALTERNATIVE BOUNDARY-LAYER THEORY FOR SEPARATED FLOW

In the classical higher order boundary-layer theory, the flow-solution is obtained by using a perturbative method. Considering Re as Reynolds number and $\varepsilon = 1/\sqrt{Re}$ a small parameter, the velocity field $\mathbf{v}$ around a solid body is represented by far-field and inner expansions:

$$\mathbf{v}=\mathbf{u}_0^{\mathrm{pf}}(\mathbf{r})+\varepsilon\mathbf{u}_1^{\mathrm{pf}}(\mathbf{r})+\ldots \text{ (far-field expansion)}, \quad \mathbf{v}=\mathbf{u}_0^{\mathrm{bl}}(l,n/\varepsilon)+\varepsilon\mathbf{u}_1^{\mathrm{bl}}(l,n/\varepsilon)+\ldots \text{ (inner expansion)}, \quad (1)$$

where $\mathbf{r}$ is the position, and l,n are the coordinates along and normal to the solid surface. The superscript 'pf' corresponds to potential flow for the far-field velocity whereas 'bl' stands for boundary-layer solution associated with the solution near the solid surface.

In the subsequent classical analysis, first $\mathbf{u}_0^{\mathrm{pf}}$ is evaluated as potential field with no-penetration at the interface. Then boundary-layer equation is solved to find $\mathbf{u}_0^{\mathrm{bl}}$ which is zero at the solid surface and matches with $\mathbf{u}_0^{\mathrm{pf}}$ in l-direction far from the body. Next, $\mathbf{u}_1^{\mathrm{pf}}$ is obtained by assuming potential flow again which properly matches with $\mathbf{u}_0^{\mathrm{bl}}$ in n-direction. This means, unlike for $\mathbf{u}_0^{\mathrm{pf}}$, the interface is not considered impermeable for $\mathbf{u}_1^{\mathrm{pf}}$. After computing $\mathbf{u}_1^{\mathrm{pf}}$, higher order boundary-layer equation is solved to calculate $\mathbf{u}_1^{\mathrm{bl}}$ using appropriate matching conditions. According to existing theory as described in any relevant text book, this is how the flow is analyzed by successive improvements.

Thus, in the classical theory, the outer field is always potential with an error near the solid surface manifested by the slip velocity. In the new theory, we have proposed a new kind of expansion which is applicable in both boundary-layer and far-field, and is capable of reducing the error in the entire solution upon addition of higher order terms. Such improved solution provides a mathematical description of separated flow by introduction of proper vorticity in the domain.

It is to be noted that contrary to the common knowledge about leading order $\mathbf{u}_0^{\mathrm{pf}}$, the higher order far-fields in classical theory do not satisfy impermeability condition at the solid surface. Hence, for these fields, the object-boundary acts as an inlet allowing small but non-zero fluid-flux in the flow-domain. This fact gives us an opportunity to introduce vorticity in the flow-solution even if the flow is inviscid in the first approximation. Our central idea is to consider a vortical field valid throughout the entire domain by introducing strong but localized vorticity which is transported by the small fluid-flux at the solid-surface. We evaluate the fluid-flux from the boundary-layer solution and determine the vorticity-flux by ensuring substantial reduction in the slip velocity.

Accordingly, we consider a slightly different expansion which is valid for the entire domain

$$\mathbf{v} = \mathbf{u}_0(\mathbf{r}) + \varepsilon(\mathbf{u}_1^{\mathrm{pf}} + \mathbf{u}_1) + \varepsilon^2(\mathbf{u}_2^{\mathrm{pf}} + \mathbf{u}_2)\cdots. \quad (2)$$

Here localized vorticity near the interface is expressed by fast-decaying $\omega_i = \mathbf{e}_z\cdot\nabla\times\mathbf{u}_i$, so that

$$\mathbf{e}_z\cdot\nabla\times\mathbf{v} = [\omega_0(\mathbf{r}/\varepsilon) + \varepsilon\omega_1(\mathbf{r}/\varepsilon) + \cdots]/\varepsilon. \quad (3)$$

It is to be noted that $\mathbf{u}_0$ is exactly the same as $\mathbf{u}_0^{\mathrm{pf}}$ far from the body and can match with $\mathbf{u}_0^{\mathrm{bl}}$ if ω_i's are properly derived. Hence, our key step is to describe the ω_i's which are very localized but much larger than unity ($\sim 1/\varepsilon$) so that, if integrated, give tangential velocity of the order 1.

First we focus on ω_0 by considering the leading order vorticity transport equation in ε

$$\mathbf{u}_0\cdot\nabla\omega_0 = 0, \qquad \Longrightarrow \qquad \nabla^2\psi = \omega_0(\psi), \quad (4)$$

where ψ is the stream-function corresponding to $\mathbf{u}_0$. Our strategy is to find the expression of $\omega_0(\psi)$ explicitly so that the function ensures a reduction in slip-velocity from the order unity to the order ε. This can be achieved by establishing a functional relation between ψ and the coordinate l where the expression for the normal flux $f(l)$ is designed to nullify the normal flux produced by $\mathbf{u}_0^{\mathrm{bl}}$:

$$\varepsilon f(l) = \mathbf{u}_0^{\mathrm{bl}}(l,\infty)\cdot\hat{\mathbf{n}} - n\frac{d}{dl}\mathbf{u}_0^{\mathrm{pf}}(l,0)\cdot\hat{\mathbf{l}} \qquad \Longrightarrow \qquad \psi = -\varepsilon\int f(l)dl = \varepsilon I(l), \tag{5}$$

where $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$ is the unit vector along n, l. As $f(l)$ is always positive, $I(l)$ is a monotonous function and eq.5 can be inverted as $l = g(\psi/\varepsilon)$. Hence, we can construct the explicit expression for $\omega_0(\psi)$ from the l-dependent slip-velocity associated with $\mathbf{u}_0^{\mathrm{pf}}$ at $n = 0$,

$$\omega_0(\psi) = -\frac{1}{2}\frac{d}{d\psi}[\hat{\mathbf{l}}\cdot\mathbf{u}_0^{\mathrm{pf}}(g(\psi/\varepsilon),0)]^2. \tag{6}$$

An inviscid but vortical field like $\mathbf{u}_0$ as defined by eqs.2 and 6 exhibits interfacial slip which is of the order ε instead of unity. The exact mathematical proof behind this conclusion is too elaborate for present discussion. However, one can physically justify it by perceiving that a right strength of vorticity can nullify the slip as well as can produce the vortical structures in the wake region.

For higher order $\mathbf{u}_i$, we need to find the next order potential field $\mathbf{u}_1^{\mathrm{pf}}$ by nullifying order ε normal flux in eq.5. This potential field has an order ε slip-velocity which is incorporated in higher order boundary-layer equation to obtain $\mathbf{u}_1^{\mathrm{bl}}$. Next, one has to solve the following vorticity equation

$$\mathbf{u}_0\cdot\nabla\omega_i + (\mathbf{u}_i^{\mathrm{pf}} + \mathbf{u}_i)\cdot\nabla\omega_0 = -\sum_{j=1}^{i-1}(\mathbf{u}_j^{\mathrm{pf}} + \mathbf{u}_j)\cdot\nabla\omega_{i-j} + \nabla^2\omega_{i-1} \tag{7}$$

and enforce the relations for corresponding normal fluid-flux and vorticity-flux to determine $\mathbf{u}_1$. Similar calculations can also be used to compute the solutions for $\mathbf{u}_2$, $\mathbf{u}_3$ etc.

Hence, efficient computation of $\mathbf{u}_i^{\mathrm{pf}}$, $\mathbf{u}_i^{\mathrm{bl}}$ and $\mathbf{u}_i$ can lead to a fast aerodynamic algorithm. Among these fields, the potential flow $\mathbf{u}_i^{\mathrm{pf}}$ is easiest to solve. However, fast evaluation of $\mathbf{u}_i^{\mathrm{bl}}$ and $\mathbf{u}_i$ is non-trivial. Fortunately, we derived a new boundary-layer solution for $\mathbf{u}_i^{\mathrm{bl}}$, and developed a simple one-dimensional evolution scheme for $\mathbf{u}_i$. We discuss such cost-reducing innovations next.

IV. NEW SEMIANALYTICAL SOLUTION FOR THE BOUNDARY-LAYER EQUATION

In a surprising discovery, we have recently solved the well-known boundary-layer equation with a novel analytical approach reducing the simulation cost substantially. By eliminating the component of $\mathbf{u}_0^{\mathrm{bl}}$ along n, we transform the common boundary-layer equation into a new convenient form with separable operator in l and n describing the streamwise component of $\mathbf{u}_0^{\mathrm{bl}}$

$$\frac{\partial u_l}{\partial l} = -\frac{g(l) - \frac{\partial^2 u_l}{\partial n^2}}{u} - \frac{\partial u_l}{\partial n}\int_0^n \frac{g(l) - \frac{\partial^2 u_l}{\partial n^2}}{u_l^2}dn. \tag{8}$$

Here u_l and g are the velocity and the free-stream pressure-gradient along l.

In the next step, we consider an expansion of u_l in terms of fast-decaying functions F_i

$$u_l(l,n) = U_l^\infty(l) + F_1(l,n) + F_2(l,n)\ldots, \tag{9}$$

where each F_i decreases quickly with increasing n representing the approach of u_l towards its free-stream value $U_l^\infty(l) = \hat{\mathbf{l}}\cdot\mathbf{u}_0^{\mathrm{pf}}(l,0)$. The decay functions satisfy the following converging criterion

$$F_i/F_{i-1} = r_i(l,n) < 1 \qquad \text{for} \qquad n > 0. \tag{10}$$

We derive a hierarchical set of equations for F_i. Though the original boundary-layer equation is non-linear, the equations for F_i are linear with explicit source terms S_i dependent on F_j with $j<i$,

$$\hat{\mathsf{L}}F_1 = 0, \qquad \hat{\mathsf{L}}F_i = S_i(F_1,F_2,\ldots,F_{i-1}) \qquad \text{for} \qquad i>1. \tag{11}$$

Here $\hat{\mathsf{L}}$ is a linear operate which depends on the free-stream velocity $U_l^\infty(l)$

$$\hat{\mathsf{L}}F = \frac{\partial}{\partial l}[U_l^\infty(l)F] - \frac{\partial^2}{\partial n^2}F - n\frac{dU_l^\infty}{dl}\frac{\partial}{\partial n}F. \tag{12}$$

The above equation is valid for any geometry of the aerodynamic body which influences operator $\hat{\mathsf{L}}$ by dictating the functional form of the free-stream velocity $U_l^\infty(l)$.

We have generalized the concept of similarity solution to derive an expression for F_1 by solving eq.11 with $i=1$. Assuming F_1 cancels the free-stream velocity at the interface $n=0$, we find that

$$F_1(l,n) = -\frac{1}{U_l^\infty(l)}\int_0^l \frac{dU_l^\infty(\lambda)}{d\lambda}\,\mathrm{Erfc}[n/s(l,\lambda)]\,d\lambda. \tag{13}$$

Here, Erfc is complementary error function and $s(l,\lambda)$ is a bivariant scaling function

$$s(l,\lambda) = \frac{1}{U_l^\infty(l)}\left[\int_\lambda^l U_l^\infty(\zeta)d\zeta\right]^{1/2} \tag{14}$$

which is a major feature of our generalized similarity solution.

The derived solution in eq.13 circumvents the problem posed by Goldstein singularity due to the inherent inclusion of essential singularities in general similarity approach. Our preliminary findings support this fact. Hence, the solution is physical in the entire flow-domain for any $U_l^\infty(l)$ representing any aerodynamic geometry. Moreover, from the expressions for F_1 and the source term S_i, the satisfaction of the convergence criteria in eq.10 can be mathematically proved. Our results show that even if we consider only $U_l^\infty(l)$ and $F_1(l,n)$ in the expansion of u_l (eq.9) neglecting the other F_i's, the obtained field can have good agreement with known boundary-layer results. For example, in Fig.1, we consider flow over a flat plate, and present Blasius's profile along with the one from our solution demonstrating a good match. Similarly, in Fig.2, we present our boundary-layer solution for flow past a cylinder. The separation point indicated by our solution agrees with the experimental observations. Also, the flow-profile and the shear-stress at the cylinder surface coincide with the corresponding known results. These solutions can be further improved by including $F_2, F_3\ldots$ obtained by very fast predictor-corrector scheme along l direction.

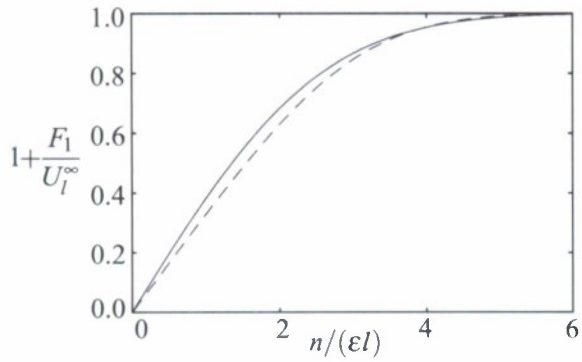

Fig.1: Preliminary validation for our boundary-layer solution in IV by considering flow over a flat plate. The solid line is for the presented approximation in eq.9 with the first two expansion terms normalized by the free-stream velocity. It is plotted as a function of the scaled distance in the normal direction. The dash-line shows the exact Blasius solution. The small error between these two curves implies a very fast convergence for the series in eq.9. Thus, we can extrapolate that the fast solution technique for the boundary-layer equation described in IV will also be effective for any geometry where analytical result is not available.

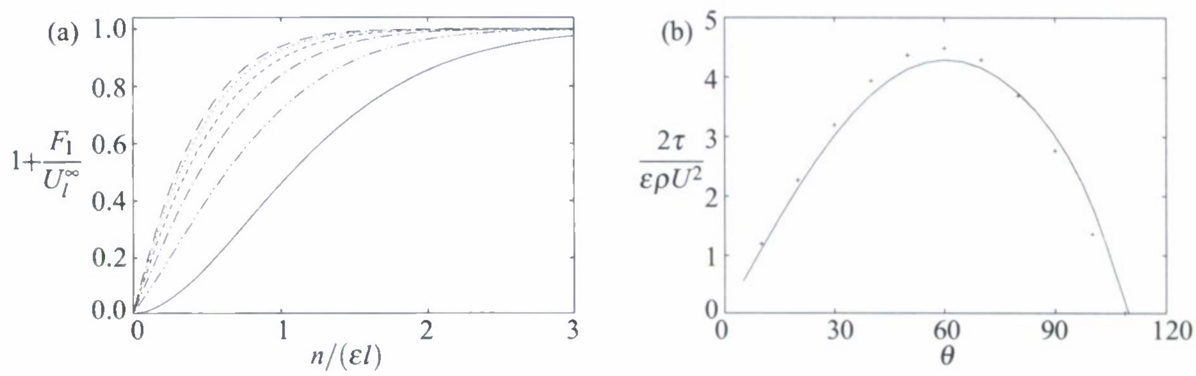

Fig.2: Preliminary validation for our boundary-layer solution in IV by considering flow past a cylinder. (a) As in Fig.1, we take into account only two terms $U_l^\infty(l)$ and F_1 in eq.9 disregarding $F_2, F_3\ldots$, and plot the normalized field as function of scaled normal distance for different angle θ measured from the leading edge with values 20^o(dashed), 40^o(dotted), 60^o(short-dash), 80^o(dash-dot), 100^o(dash-dot-dot). The solid line represents 108.8^o, the known point of separation where the computed velocity-gradient at the wall is very small. The approximate analytical solution is in very good agreement with existing results [108] implying fast convergence of the series in eq.9. (b) The solid line shows the non-dimensional wall shear-stress as function of θ for only F_1, whereas the '+' points are for known stress-values [108]. The two sets of results have striking agreement. We overpredict the separation point only by 2^o which can be corrected by adding $F_2, F_3\ldots$

V. SCHEME TO SOLVE VORTICAL FIELD

The other components of our algorithmic innovation is to formulate an efficient technique to evaluate the vortical flow-field $\mathbf{u}_i$. For brevity, here we only focus on evaluation of $\mathbf{u}_0$.

For efficient computation of $\mathbf{u}_0$, the domain is divided into region of non-zero vorticity and region of potential flow. The two distinct regions are separated by a streamline of zero vorticity identified by superposing potential fields $\mathbf{u}_0^{\text{pf}}$ and $\mathbf{u}_1^{\text{pf}}$. A streamline originates from the upstream stagnation point, separates from the body, and continues to infinity in the downstream, when a field like $\mathbf{u}_1^{\text{pf}}$ with non-zero wall-flux is superposed on $\mathbf{u}_0^{\text{pf}}$. Such streamline contains zero-vorticity due to the flow-behavior at the stagnation point. In our opinion, the zero-vorticity line defines the

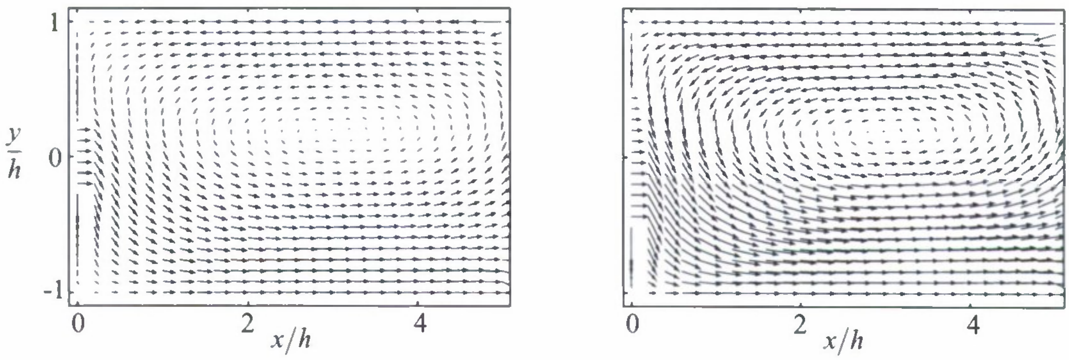

Fig.3:Vortical inviscid flow due to incoming vorticity in an expansion chamber with different inlet-widths computed by the outlined evolution scheme in V.

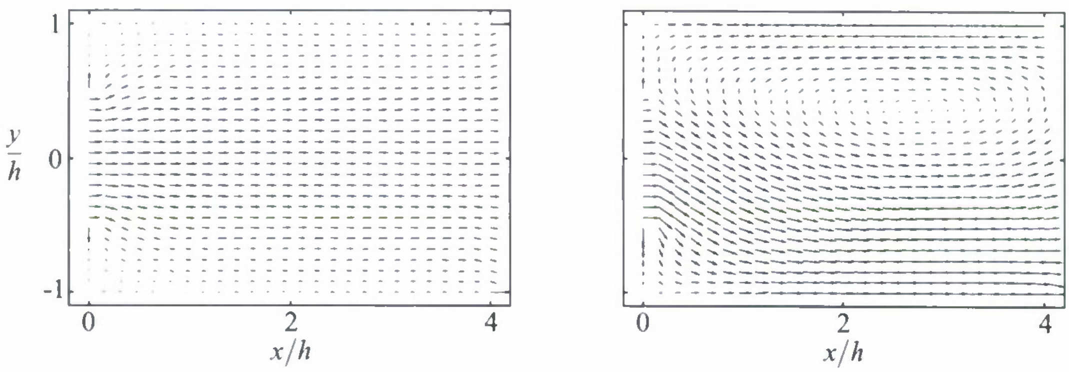

Fig.4: Same as Fig.3 only inlet velocity profile is different instead of inlet width.

wake region by demarcating the vortical and potential field. We compute $\mathbf{u}_0$ only in the vortical subdomain because rest of the flow is simple superposition of $\mathbf{u}_0^{\mathrm{pf}}$ and $\mathbf{u}_1^{\mathrm{pf}}$. Due to the property of zero-vorticity streamline, there is smooth transition between the potential and vortical regions.

Once the zero-vorticity streamline separating the vortical and potential region is identified, we obtain $\mathbf{u}_0$ by determining the velocity on the streamlines and their shape in the vortical subdomain. Due to the wall-flux in $\mathbf{u}_1^{\mathrm{pf}}$, all streamlines in vortical subdomain originates from the interface and continues to infinity in the downstream. One can form evolution equations along the streamlines to find their curvature and velocity by exploiting known fluid-flux and vorticity-flux given by eqs.5 and 6, respectively. Thus, this simple and highly efficient scheme can solve $\mathbf{u}_0$ very quickly.

In Figs.3 and 4, we present a test problem which demonstrates the vortical structures in inviscid flow constructed by the aforementioned scheme. Here, instead of eq.6, we assume a hypothetical linear relation between the vorticity and the stream function. We construct the vortical field inside a rectangular expansion cavity. Though the system only corresponds to a mathematical problem without any significant physical interpretation, one can see the similarity between circulating vortices captured by our algorithm and eddies in the wake of a separated flow.

VI. BRIEF DESCRIPTION OF THE AERODYNAMIC ALGORITHM AND COST ESTIMATION

We build the fast aerodynamic algorithm based on 1) the new flow-separation theory, 2) the semianalytical boundary-layer solution, and 3) the evaluation scheme for the vortical field. The approach works for any geometry of the aerodynamic body. Only requirement for the method is availability of the leading order potential field $\mathbf{u}_0^{\mathrm{pf}}$ which is easy to determine for a given geometry.

When $\mathbf{u}_0^{\mathrm{pf}}$ is known, the fast method finds the flow-field and the wall-stresses as well as the lift and drag on the body in a six-step computation. These six steps are described below.

• *Step 1*:– We determine the free-stream velocity $U_l^\infty(l)$ for the tangential velocity u_l in boundary-layer from $\mathbf{u}_0^{\mathrm{pf}}$. Then, we use the semianalytical method outlined in IV to find the function F_1 from eq.13 and other functions $F_2, F_3, \ldots$ by solving eq.11 with Euler predictor-corrector method. As a result, u_l can be evaluated from the expansion eq.9.

• *Step 2*:– The normal component of the boundary-layer velocity is obtained from u_l. Then, the interfacial fluid-flux and vorticity-flux as in eqs.5 and 6 are calculated using the solution for $\mathbf{u}_0^{\mathrm{bl}}$.

• *Step 3*:– We obtain the higher order potential field $\mathbf{u}_1^{\mathrm{pf}}$ which corresponds to the wall fluid-flux.

• *Step 4*:– The evaluated fluid-flux and vorticity-flux at the wall as well as the superposed potential fields $\mathbf{u}_0^{\mathrm{pf}}$ and $\mathbf{u}_1^{\mathrm{pf}}$ are used in the scheme outlined in V to compute $\mathbf{u}_0$.

• *Step 5*:– We combine $\mathbf{u}_0$ with $\mathbf{u}_1^{\mathrm{pf}}$ to nullify the normal fluid-flux at the solid surface.

• *Step 6*:– The previous step introduces an interfacial slip-velocity of the order ε. This slip-velocity can be considered as the free-stream velocity for the higher order boundary-layer field $\mathbf{u}_1^{\mathrm{bl}}$. Hence, the step 1 to 5 can be repeated to solve for the next order fields in ε. Thus, one can continue until an arbitrary order and achieve an arbitrary accuracy in the process.

Among steps 1 to 5, only time consuming ones are step 1 and 4. If we want to limit the relative simulation-error to less than 1%, we have to include F_1, F_2, F_3 and F_4 in step 1. The computation cost for F_1 is at least ten times more than other F_i's. In our analysis with a $2.4GHz$ machine, we have checked the time for evaluation of F_1 to be 0.02second. Our estimate of simulation time for next four F_i's together is 0.01second. Also, we have seen that step 4 takes approximately 0.01second. Therefore, we can conclude that evaluation of velocity field of certain order in ε takes less than 0.05second. For less than 1% error, first two to three leading order fields need to be computed. Thus, the fast aerodynamic scheme can provide a very accurate solution in about 0.1second. Any other numerical method takes at least more than a minute to solve the equivalent problem. Hence, the hundredfold increase in efficiency can be utilized for exploration of new designs and study of new aerodynamic systems.

VII. SUMMARY AND CONCLUSION

In this project, we have formulated an alternative boundary-layer theory. This new analysis will be able to mathematically describe flow-separation unlike the classical theory.

In our research, we have partially validate the developed theory. For example, we can reproduce Blasius profile for a boundary-layer on a flat plate. Similarly, we can accurately analyze separated flow from a cylinder, and find the correct wall shear-stress. We were also able to determine the point of separation where the wall shear-stress vanishes. Hence, we can conclude that our approach has considerable potential to account for flow-separation.

The effective description of separated flow can potentially lead to a fast simulation-algorithm for aerodynamic computation. Our estimate predicts that this semianalytical scheme will compute the lift and drag on an aerodynamic body in less than 0.1sec with less than 1% relative error. This is more than hundredfold increase over current simulation-efficiency.

The enhanced efficiency will enable hitherto impossible exploration of new designs for maximization of the lift to drag ratio. In the future, this will revolutionize aviation technology by the development of bio-inspired aviation mechanism and other novel systems. Such improvements will help in energy-savings and pollution control by reducing fuel consumptions.

We know that more work should be done on this very powerful theory which we have developed during the funding period of eight months from April 2008 to November 2008. We hope to receive further funding in the future to continue this research to its proper conclusion.

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